Conservation of cultural heritage structures in seismic regions

P.B. Lourenço
Department of Civil Engineering, University of Minho, Azurém, 4810-058 Guimarães, Portugal. Tel. +351 253 510200. Fax. +351 253 510217. E-mail: pbl@civil.uminho.pt. Webpage: www.civil.uminho.pt

ABSTRACT: The paper addresses different aspects related to the conservation of cultural heritage structures, with a focus on: (a) Behavior of masonry components under cyclic loading (tension, compression and shear); (b) Behavior of stone masonry shear walls under cyclic loading; (c) Behavior of dry masonry blocks and structures under dynamic loading; (d) Behavior of masonry arches strengthened with FRP; (e) Possibilities of numerical analysis at the laboratory and engineering levels; (f) An European Commission funded research project on reducing seismic vulnerability of cultural heritage buildings.

1 INTRODUCTION

Modern societies understand built cultural heritage as a landmark of culture and diversity. Only during the last decades the idea that old and ancient buildings could be restored and reused became appealing for the market. In fact, the present policy is not only to preserve but also to make buildings and the historic part of the cities alive, functioning and appealing to the inhabitants and to the tourists.

Nevertheless due to the effects of aggressive environment (earthquakes, soil settlements, traffic vibrations, air pollution, etc.) and to the fact that many old buildings and historic centers were not subject to continuous maintenance, a large part of this heritage is affected by structural problems that menace the safety of buildings and people.

European countries have developed a valuable experience in conservation and restoration. In recent years, large investments have been concentrated in this field, leading to impressive developments in the areas of inspection, non-destructive testing, monitoring and structural analysis of historical constructions. These developments allow for safer, economical and more adequate remedial measures.

Being earthquakes a major source of destruction of cultural heritage buildings, this paper focus on recent advances related to their conservation.

2 EXPERIMENTAL ISSUES

Masonry is a heterogeneous material that consists of units and joints. Units are such as bricks, blocks, ashlars, adobes, irregular stones and others. Mortar can be clay, bitumen, chalk, lime/cement based mortar, glue or other. The huge number of possible combinations generated by the geometry, nature and arrangement of units as well as the characteristics of mortars raises doubts about the accuracy of the term “masonry”. Nevertheless, most of the advanced experimental research carried out in the last decades has concentrated in brick / block masonry and its relevance for design. Accurate modeling requires a thorough experimental description of the material, see Lourenço (1998) and Cur (1997).

2.1 *Properties of unit and mortar*

The properties of masonry are strongly dependent upon the properties of its constituents. Compressive strength tests are easy to perform and give a good indication of the general quality of the materials used. Experiments about the uniaxial post-peak behavior and about the biaxial behavior of bricks and blocks are less common in the literature, together with tests on cyclic behavior. Next, some results for clay bricks under uniaxial compression are briefly reviewed (Oliveira et al. 2005). A series of unloading-reloading cycles were performed in clay specimens, particularly in the post-peak region, to acquire data about stiffness degradation and energy dissipation. The experimental set-up, testing conditions and typical stress-strain diagrams are illustrated in Figure 2. The response indicates an important and monotonic decrease in Young’s modulus in the post-peak regime, associated with damage growth in the material.

With respect to the tensile strength of the masonry unit, extensive information on the tensile strength and fracture energy of units can be found in Lourenço et al. (2005) and Vasconcelos (2005), see Figure 2. The difficulties in relating the tensile strength of the unit to its compressive strength are well known, not only due to the different shapes of the units but also to the different materials.

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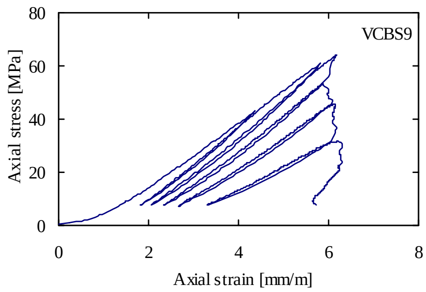

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Figure 1. Aspects related to the cyclic behavior of masonry units under uniaxial compression: (a) cylindrical brick specimen under testing conditions, and (b) typical stress-strain diagram.

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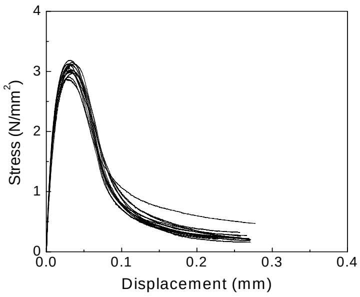


(b)

Figure 2. Aspects related to the behavior of masonry units under tension: (a) notched stone specimen under testing conditions, and (b) typical stress-strain diagrams.

2.2 *Properties of the interface*

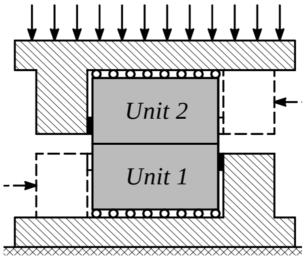
Bond between unit and mortar is often the weakest link in masonry assemblages. The non-linear response of the joints, which is then controlled by the unit-mortar interface, is one of the most relevant features of masonry behavior. Two different phenomena occur in the unit-mortar interface, one associated with tensile failure (mode I) and the other associated with shear failure (mode II). Different test set-ups have been used for the characterization of the tensile behavior of the unit-mortar interface. For the purpose of numerical simulation, direct tension testing should be adopted because it allows for the full representation of the stress-displacement diagram and yield the correct strength value. No tests seem to be reported with respect to the behavior of the interface under cyclic tension.

Adequate characterization of masonry shear behavior under cyclic loading is given in Lourenço & Ramos (2004), as shown in Figure 3. The experimental set-up has been designed so that the bending effects associated with shear testing are minimized. The vertical confining pressure is kept constant while the test is carried out under horizontal displacement control. Almost zero dilatancy has been found during each cycle. The tests indicate that the shear inelastic deformation is fully plastic (or irreversible).

2.3 *Properties of the composite material*

The compressive strength of masonry in the direction normal to the bed joints has been traditionally regarded as the sole relevant structural material property. Since long it has been accepted by the masonry community that the difference in elastic properties of the unit and mortar is the precursor of failure, but this seems hardly correct (Pina-Henriques and Lourenço, 2006) and Figure 4. Uniaxial compression tests in the direction parallel to the bed joints have received substantially less attention from the masonry community.

Next, some results for masonry specimens under uniaxial compression (Oliveira et al., 2002) are briefly reviewed. A series of unloading-reloading cycles were performed, particularly in the post-peak region, to acquire data about stiffness degradation and energy dissipation. The typical failure and stress-strain diagrams are illustrated in Figure 6. Stress-strain curves exhibited a pre-peak bilinear behavior, which has been reported by other authors. An initial linear branch was followed by another branch up to near the peak, with lower stiffness and greater development. The response clearly indicates an important and monotonic decrease in Young's modulus in the post-peak regime, associated with damage growth in the material.



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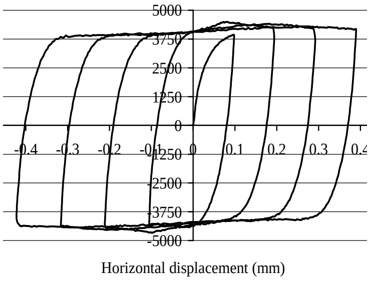

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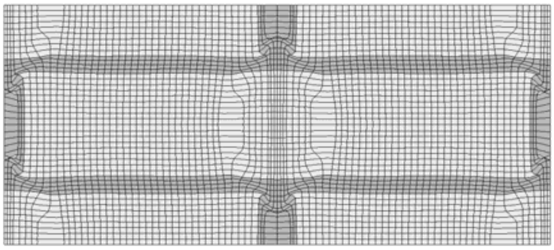
Figure 3. Aspects related to the cyclic behavior of masonry joints under shear: (a) specimen under testing conditions, and (b) typical stress-strain diagram.



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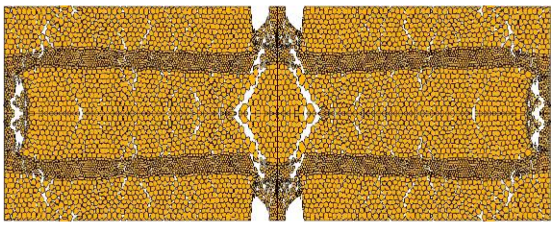

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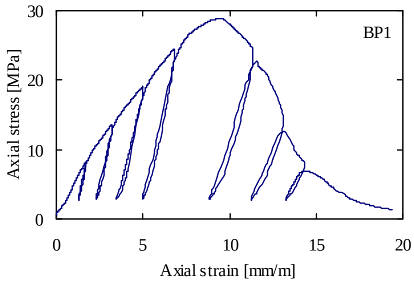
Figure 4. Simulation of a masonry representative volume under compression: (a) continuum model, and (b) particulate model. The differences found in terms of simulated compressive strength are up to 30%.

(a)



(b)

Figure 5. Aspects related to the cyclic behavior of masonry specimens under uniaxial compression: (a) typical failure of masonry specimen and (b) typical stress-strain diagram.

2.4 *Stone masonry shear walls*

Although traditional historic masonry walls can be viewed as unsuitable structures to undergo seismic actions, they, in fact, exist and frequently represent the major structural elements of ancient buildings. Brick unreinforced masonry walls have been widely studied both from experimental and numerical point of view, but scarce experimental information is available for stone masonry walls. Therefore, a comprehensive testing program was started at University of Minho, aiming at increasing the insight about the behavior of typical ancient masonry walls under cyclic loading (Vasconcelos 2005). Besides the strength and stiffness characterization, information about nonlinear deformation capacity was obtained in terms of ductility factors and lateral drifts, which represents a step forward for the new concepts of performance based design.

Regular and irregular stones have been adopted, see Figure 6. Although no significant differences were found in terms of strength and lateral stiffness among the distinct types of walls, low strength mortared masonry walls exhibit markedly higher level of energy dissipation when compared with dry stacked masonry.

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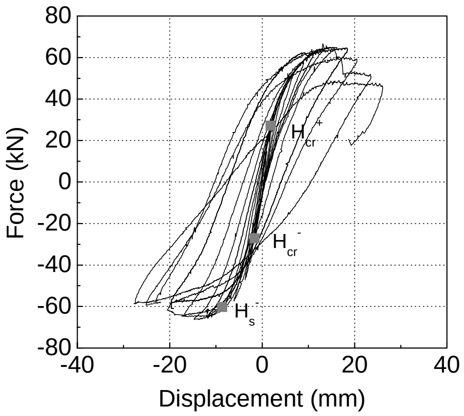


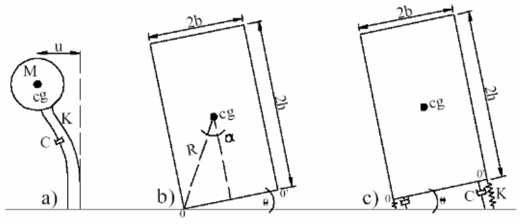
(b)

Figure 6. Behavior of stone masonry walls with different bond: (a) failure modes and (b) selected force-displacement diagram.

2.5 *Dry blocky stone masonry structures*

The behavior of masonry can often be associated with dry blocky structures, which feature zero tensile strength in the joints but horizontal tensile strength and shear strength due to frictional effects. Limit analysis simulations are often used in practice for safety assessment and strengthening design. In order to extend limit analysis formulation to include dynamics and in order to study out of plane seismic behavior of masonry walls, another comprehensive testing program was set-up at University of Minho and National Laboratory of Civil Engineering (LNEC, Portugal).

Figure 7 illustrates the details of the testing program, including simplified analysis models, structures under analysis and results.



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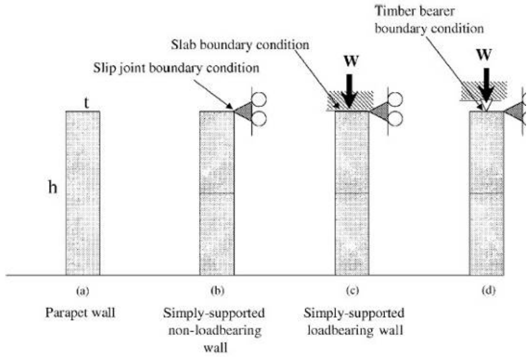

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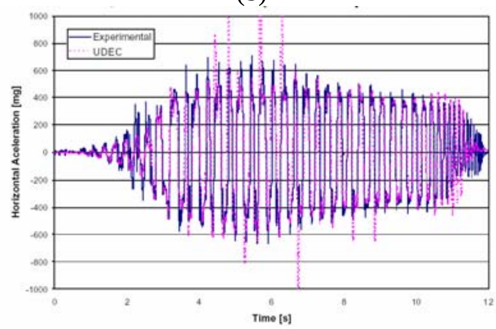
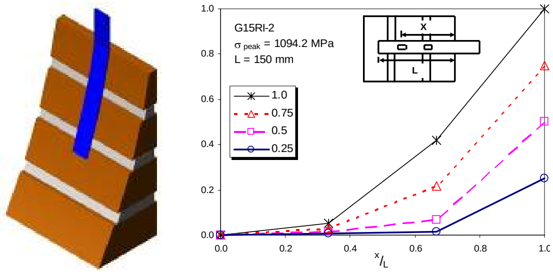


(c)

Figure 7. Dynamic behavior of blocky stone structures: (a) simplified models for analysis, (b) possible out-of-plane conditions for masonry walls, and (c) typical experimental / numerical results for hanning sinusoidal forced vibration.

2.6 *FRP Reinforced masonry arches*

Among the materials used to repair or upgrade civil engineering structures, there has been an increasing interest devoted to the use of FRP (fiber-reinforced polymer) composites in the form of bonded surface reinforcements. A set of eight arches, built with traditional low strength materials, have been tested under a monotonic vertical load applied at the quarter span, with different positions for the strengthening. In addition, also the bond between masonry and FRP have been characterized in a testing program being finalized, see Figure 8.



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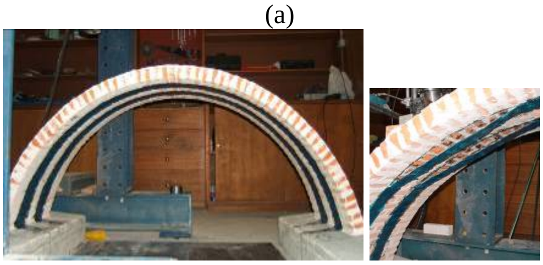


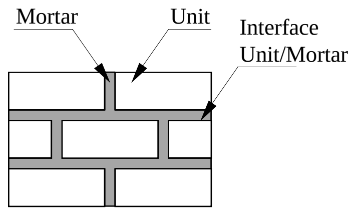
(b)

Figure 8. Traditional brick masonry strengthened with FRP: (a) bond tests with different masonry curvatures; (b) arch tests under point load.

3 NUMERICAL ISSUES

Depending on the level of accuracy and the simplicity desired, it is possible to use different modeling strategies shown in Figure 9. Micro-modeling studies are necessary to give a better understanding about the local behavior of masonry structures. This type of modeling applies notably to structural details. Macro-models are applicable when the structure is composed of solid walls with sufficiently large dimensions so that the stresses across or along a macro-length will be essentially uniform. Clearly, macro modeling is more practice oriented due to the reduced time and memory requirements as well as a user-friendly mesh generation.

Linear elastic analysis can be assumed a more practical tool, even if the time requirements to construct the finite element model are the same as for non-linear analysis. But, such an analysis fails to give an idea of the structural behavior beyond the beginning of cracking. Due to the low tensile strength of masonry, linear elastic analyses seem to be unable to represent adequately the behavior of historical constructions.



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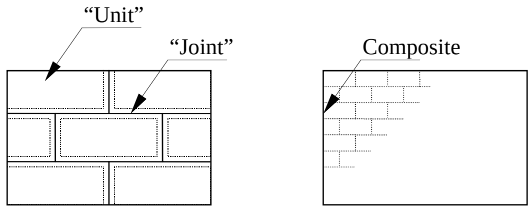


(b) (c)

Figure 9. Modeling strategies: (a) masonry specimen, (b) micro-modeling and (c) macro-modeling.

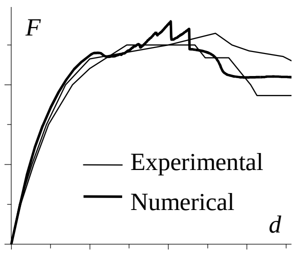
3.1 *Discontinuum models (Micro-modeling)*

This kind of analysis is particularly adequate for small structures, subjected to states of stress and strain strongly heterogeneous, and demands the knowledge of each of the constituents of masonry (unit and mortar) as well as the interface. In terms of modeling, all the non-linear behavior can be concentrated in the joints and in straight potential vertical cracks in the centerline of all units. In general, a higher computational effort ensues, so this approach still has a wider application in research and in small models for localized analysis. Applications can be carried out using finite elements, discrete elements or limit analysis.

The salient characteristics of discrete elements are: (a) rigid or deformable (combined with the finite element method) blocks; (b) connection between vertexes and sides / faces; (c) interpenetration possible, integration of the equation of motion (explicit formulation); (d) real damping coefficient (dynamic problem) or artificially high damping (static solution). The main advantages of the technique are the adequacy of the formulation for large displacements (contact update), and independent meshes for each deformable block. The main disadvantages are that a high number of contact points is needed for accurate representation of tractions in the interface, and the time requirements are rather high for large meshes, namely for 3D problems.

The salient characteristics of limit analysis are: (a) rigid blocks; (b) interpenetration not allowed; (c) mathematical formulation that leads to an optimization problem (linear or non-linear). The main advantages of the technique are adequate formulation for design problems (requires a low number of parameters) and fast analysis. The main advantages are that only the collapse load and mechanism can be obtained, tensile strength cannot be included in the analysis, and the introduction of the loading history remains a challenge.

A complete micro-model must include all the failure mechanisms of masonry, namely, cracking of joints, sliding over one head or bed joint, cracking of the units and crushing of masonry, as in Lourenço & Rots (1997) and Oliveira & Lourenço (2004). Figure 10 shows the results of modeling a shear wall with an initial vertical pre-compression pressure. The horizontal force F drives the wall to failure, keeping the top and bottom boundaries fully constrained, and produces a horizontal displacement d at top. Initially, two horizontal cracks develop at the top and bottom of the wall but at failure a diagonal stepped crack and crushing of the compressed toes are found. A complete discussion of the numerical results has been given in (Lourenço & Rots 1997).



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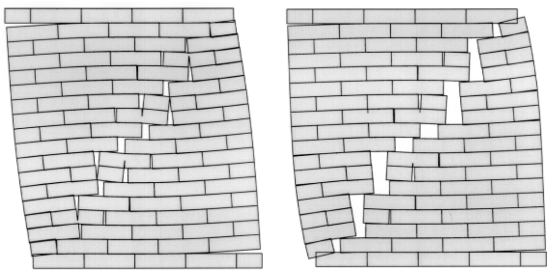

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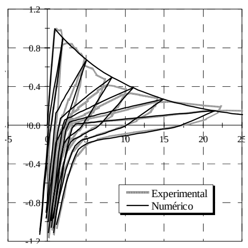
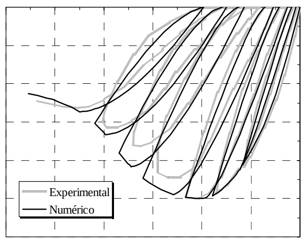
Figure 10. Results for an analysis of a shear wall (micro-modeling): (a) force-displacement diagram and (b) deformed meshes at peak and ultimate load.

The extension of the above model to include cyclic behavior is given in Oliveira & Lourenço (2004). To include non-linear unloading/reloading behavior in an accurate fashion, new yield surfaces are introduced in the above monotonic model. In the proposed model, the motion of the unloading surfaces is controlled by a mixed hardening law. By adopting appropriate evolution rules, it is possible to reproduce non-linear behavior during unloading, see Figure 11. The recent experimental work in the cyclic behavior of interfaces described in the previous chapter has shown some important characteristics, namely stiffness degradation in tension and compression regimes, residual relative displacements at zero stress, absence of stiffness degradation in direct shear, and complete crack closing under compressive loading. The available experimental results concerning the cyclic behavior of interfaces suggest that: (a) Elastic behavior constitutes a satisfactory approach for shear unloading/reloading behavior; (b) Elastic unloading/reloading is not an appropriate hypothesis for tensile and compressive loading since observed experimental behavior cannot be simulated accurately, namely stiffness degradation and crack closing/reopening, which clearly exhibit non-linear behavior.





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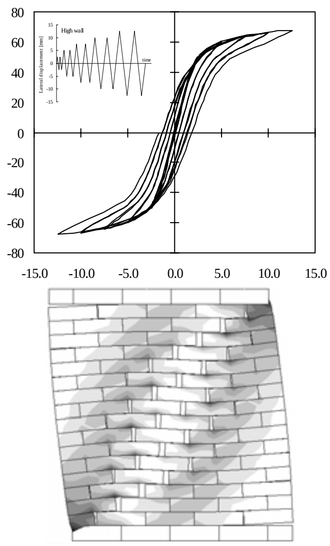


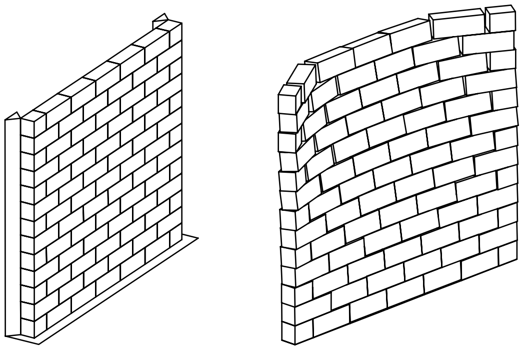
(c)

Figure 11. Behavior for an interface model extended to cyclic formulation: (a) tension-compression, (b) compression and (c) shear walls.

The drawback of using non-linear finite element analysis in practical situations might include: (a) requirement of adequate knowledge of sophisticated non-linear processes and advanced solution techniques by the practitioner; (b) comprehensive mechanical characterization of the materials; and (c) large time requirements for the construction of the finite element model, for performing the analyses themselves and for reaching proper understanding of the results significance.

Limit analysis combines, on one hand, sufficient insight into collapse mechanisms, ultimate stress distributions (at least on critical sections) and load capacities, and on the other hand, simplicity to be cast into a practical computational tool. In addition, the number of necessary material parameters is low.

Figure 12 illustrates results using advanced solution procedures for non-linear optimization problems, with a constitutive model that incorporates non-associated flow at the joints and a novel formulation for torsion, Orduña & Lourenço (2005)



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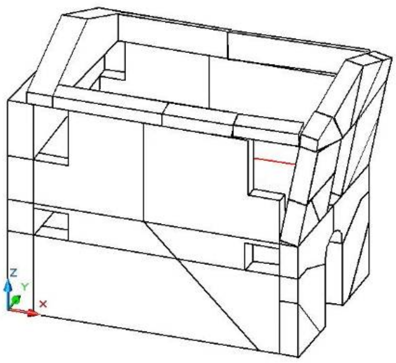


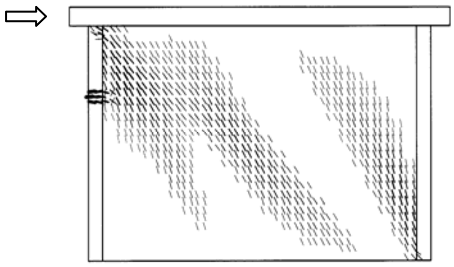
(b)

Figure 12. Results for different analyses (micro-modeling, using limit analysis): (a) panel subjected to out-of-plane failure and (b) simplified analysis of a complete building with macro-blocks.

3.2 *Finite element models for continua (Macro-modeling)*

Only a reduced number of authors tried to develop specific models for the analysis of masonry structures, always using the finite element method. Formulations of anisotropic quasi-brittle behavior consider, generally, different inelastic criteria for tension and compression. The model introduced in Lourenço et al. (1998) and extended to accommodate shell behavior (Lourenço 2000), combines the advantages of modern plasticity concepts with a powerful representation of anisotropic material behavior, which includes different hardening/softening behavior along each material axis.

Figure 13 shows the results of modeling a shear wall with an initial vertical pre-compression pressure and a wall panel subjected to out of plane failure.



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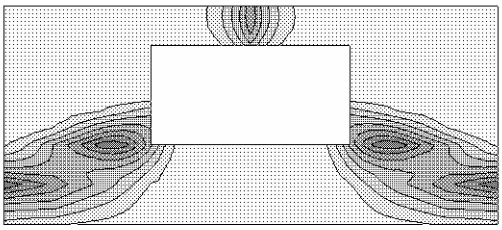

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Figure 13. Results for different analyses (macro-modeling): (a) shear wall and (b) panel subjected to out-of-plane failure.

4 EU-INDIA CONTRACT "IMPROVING THE SEISMIC RESISTANCE OF CULTURAL HERITAGE BUILDINGS"

The main objective of this project is the development of a social and economic argument, at Indian-European level, to support an earthquake protection innovative program for cultural heritage masonry buildings at risk. This will consider cultural heritage buildings / monuments in an earthquake prone area in India, identify seismic input scenarios and specific vulnerability features, and study advanced upgrading and strengthening techniques, based on four case studies (see Figure 14). The Plan of Action is based on a multidisciplinary approach, entailing aspects of risk analysis, in situ survey and monitoring, numerical analyses and the design/application of innovative strengthening strategies. The objective is to devise strengthening strategies that, based on thorough knowledge of the traditional craft and material, can use modern materials and techniques to prevent vibration borne damage to the structures and to the decorative apparatus.

The proposal mainly focuses on: (a) The identification of preventative measures that can be implemented to improve the earthquake resistance of historic masonry Cultural/Historical Buildings (CHBs) and Cultural Heritage in general; (b) Definition and application of optimal modeling strategies for determining the load bearing capacity of historic structures before and after repair; (c) Cost/benefit analysis of the proposed procedures taking into account the different levels of complexity and of disposable budget; (d) Set up of a comprehensive database of traditional local technologies for construction and repair; (e) Full conservation design for three case studies selected in Europe and India; (g) The interchange of knowledge between European and Indian experts.

This implies a better understanding and enhancement of the inherent earthquake-resistant characteristics of CHBs achieved through compared vulnerability analysis, in situ monitoring of real cases and numerical simulation.

The activities included in the project are: (a) Inventory of monuments at risk; (b) Seismic activity evaluation and site effects; (c) Conference in Padova, Italy; (d) In situ tests and monitoring; (e) Evaluation and design of strengthening; (f) Definition of guidelines; (g) Dissemination; (h) Conference in New Delhi, India.

4.1 *Highlights of results*

One objective was to evaluate the possibility to adopt simple indexes related to geometrical data as a first (very fast) screening technique to define priority of further studies with respect to seismic vulnerability. These techniques are to be used without actually visiting the buildings, being therefore not accurate. It is expected that the geometrical indexes could detect cases in serious risk and, thus, define priority of additional studies in countries/locations without recent moderate or severe earthquakes.

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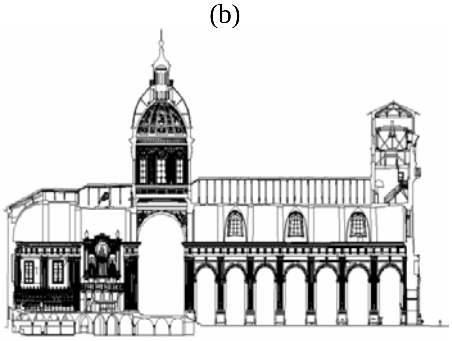

(c)

(d)

Figure 14. Case studies: (a) Monastery of Jerónimos, Lisbon, Portugal – World Heritage Monument; (b) Cathedral of Majorca, Spain; (c) Qutub Minar, New Delhi, India – World Heritage Site; (d) Cathedral of Reggio Emilia.

Forty-four buildings from Portugal, Spain and Italy have been selected and analyzed considering three in-plane indexes and three out-of-plane indexes. The proposed indexes of monuments located in different seismic areas are compared with the respective seismic hazard, i.e. the peak ground acceleration (PGA), defined for a 10% probability of exceedance in 50 years for a rock-like soil, corresponding to a return period of 475 years. The recognition of the likely existence of a correlation between structural characteristics and seismic hazard is, therefore, sought. An example of the values computed for one in-plane index (in plan area ratio) and the proposed threshold is shown in Figure 15, see University of Minho (2005) for details.

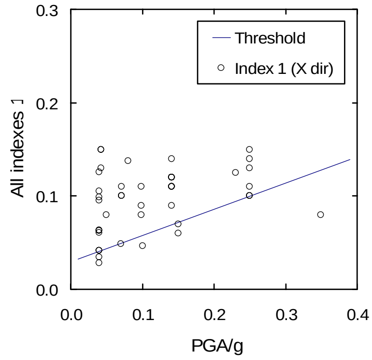

Figure 15. Typical example of the results obtained for the entire sample and the proposed threshold.

A second objective was to apply a common methodology to four case studies in different countries. In Portugal, Monastery of Jerónimos, Lisbon, has been adopted as case study. Monastery of Jerónimos is, probably, the crown asset of Portuguese architectural heritage dating from the 16th century. The monumental compound has considerable dimensions in plan, more than 300 × 50 m^2, and an average height of 20 m (50 m in the towers). The monastery evolves around two courts. The construction resisted well to the earthquake of November 1, 1755. Later, in December 1756, a new earthquake collapsed one column of the church that supported the vaults of the nave and resulted in partial ruin of the nave. In this occasion also the vault of the high choir of the church partially collapsed.

The Gothic style was lately introduced in Portugal, incorporating a specific national influence. The so-called “Manueline” style (after King D. Manuel I), exhibits a large variety of architectural influences and erudite motives. An interesting aspect appears in the 16th century, when the traditional three naves churches start to be replaced by a configuration with small difference in height for the naves. Here, the vault springs from one external wall to the other, supported in slender columns that divide almost imperceptibly the naves. From the traditional art, only the proportions and roof remain, being the concepts of space and structure novel. The fusion of the naves in the present Church, see Figure 16, is more obvious than in other manifestations of spatial Gothic. For this purpose, arches are no longer visible, the slightly curved vault comprises a set of ribs and the fan columns reduce effectively the free span. Additional information about the church and the vault can be found in Genin (2001).

Figure 16. Monastery of Jerónimos: view of the nave and choir.

The church has considerable dimensions, namely a length of 70 m, a width of 40 m and a height of 24 m. The plan includes a single bell tower (south side), a single nave, a transept, the chancel and two lateral chapels. In order to assess the safety of the church, several in situ tests have been carried out: (a) three-dimensional survey of the church; (b) sonic and GPR tests in the columns to assess the integrity; (c) radar investigation to detect the thickness of the masonry infill in the vault and pier [10]; (d) removal of the roof, visual inspection, bore drilling, metal detection and chemical analysis of materials; (e) dynamic identification, see Figure 17 for examples.

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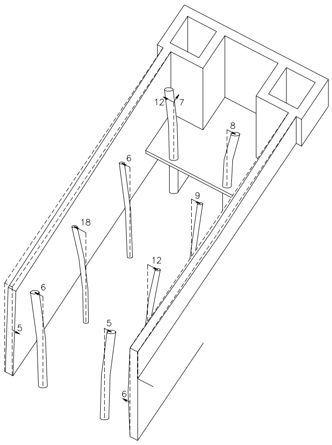

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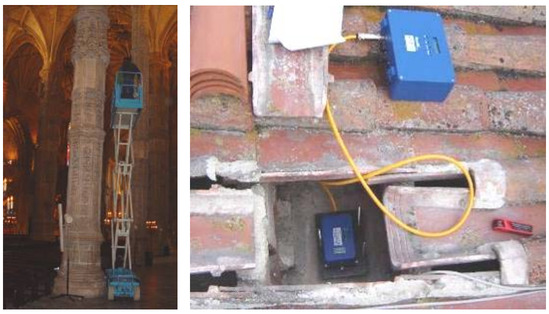

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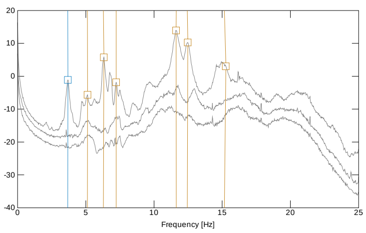

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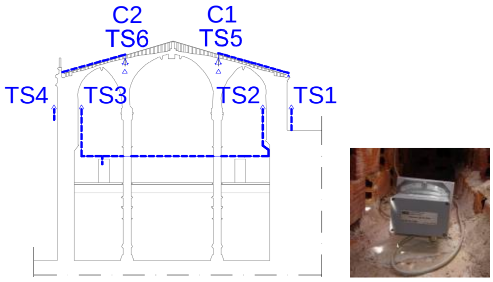

(e)

Figure 17. In situ testing and monitoring for Monastery of Jerónimos: (a) inspection of the vault nave; (b) survey of the columns; (c) radar inspection and ambient vibration acquisition; (d) dynamic identification; (e) static and dynamic monitoring.

Advanced structural analysis was considered in order to quantify the seismic vulnerability. Different models have been used to study the behavior of the compound and of the church, see Figure 18. In the complete model of the compound only the very large openings were considered and the geometry of the model was referred to the average surfaces of the elements. All the walls, columns, buttresses, vaults and towers were included in the model, with the exception of a few minor elements. The finite element mesh is predominantly rectangular and structured, but, for the towers and local refinements, triangular finite elements are also adopted. All elements possess quadratic displacement fields. The mesh includes around 8000 elements, 23500 nodes and 135000 degrees of freedom. The time necessary for total mesh generation, including definition of supports, loads and thicknesses, can be estimated in three months. A push-over analysis with zero tensile strength indicated that the towers of the Museum are the critical structural elements featuring, at collapse, displacements of around 0.10 m and cracks of around 0.01 m. Smaller cracks are also visible in the church. The analyses indicate that the monastery is a safe construction, with respect to the wall behavior. As the vaults were not properly considered, a conclusion regarding the safety of the vaults (thus, of the church) is impossible.

In order to better study the church, a more refined model was adopted for the main nave, including the structural detail representative of the vault. Symmetry, conservative, boundary conditions have been incorporated. Therefore, the model represents adequately the collapse of the central-south part of the nave. The model includes three-dimensional volume elements, for the ribs and columns, and curved shell elements, for the infill and stones slabs. The external (south) wall was represented by beam elements, properly tied to the volume elements. The supports are fully restrained, being rotations possible given the non-linear material behavior assumed. All elements have quadratic interpolation, resulting in a mesh with 33335 degrees of freedom.

Fig. 18d illustrates the load-displacement diagrams for the vault key and top of the column. Here, the load factor represents the ratio between the self-weight of the structure and the applied load, meaning that the ultimate load factor is equivalent to the safety factor of the structure. It is possible to observe that the response of the structure is severely nonlinear from the beginning of loading, for the nave, and from a load factor of 1.5, for the column. The behavior of the nave is justified by the rather high tensile stresses found in the ribs, using a linear elastic model. The collapse of the columns is due to the normal and flexural action. The safety factor is 2.0, which is relatively low for this type of structures. The stresses are bounded in tension and compression, meaning that cracking and crushing occurs. The pairs of transverse ribs that connect the columns (in the central part of the structure) exhibit significant cracking, as well as the infill in the same area. Additional cracking, less exuberant and more diffused, appears in the central octagon defined by the capitals of the four columns. Such cracking occurs at the key of the octagon and in the longitudinal ribs, which confirms the larger displacements of the vault and the bidirectional behavior of the vault.

5 CONCLUSIONS

Significant knowledge is available in the context of modern testing and advanced analysis of masonry structures. Constraints to be considered in the use of advanced modeling are the cost, the need of an experienced user / engineer, the level of accuracy required, the availability of input data, the need for validation and the use of the results. Obtained results are usually important for understanding the structural behavior of the constructions. But, as a rule, advanced modeling is only necessary in practice to understand the behavior and damage of (complex) constructions and to assist in the definition of rational safety assessment rules, based on a reliable and economical numerical laboratory. The key message of the paper is that research and innovation are strongly needed to assess the vulnerability of existing constructions and to define economical rational design rules. Without this, the ancient household and the preservation of the architectural heritage remain at risk. For this purpose, an example of recent results involving a large project funded by the European Commission is presented.

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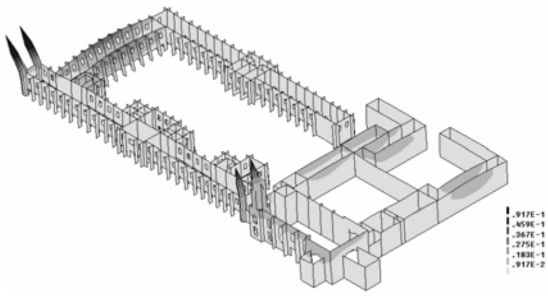

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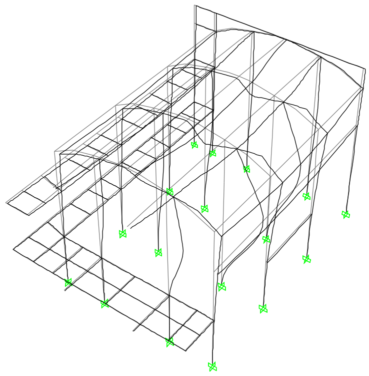

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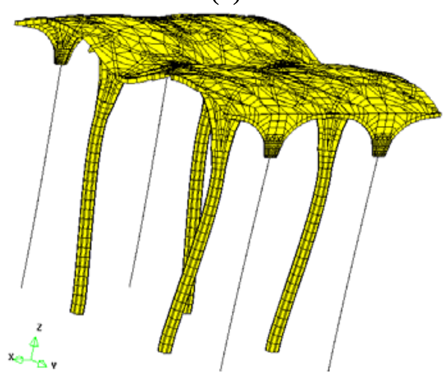

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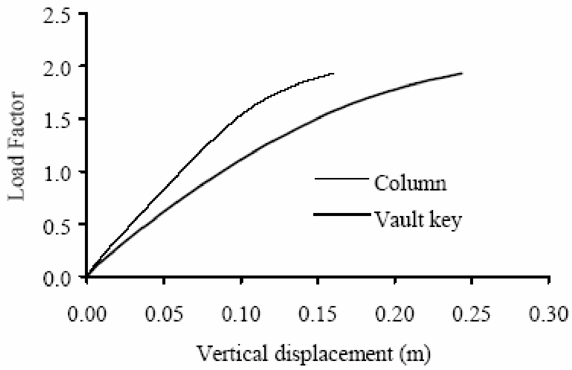

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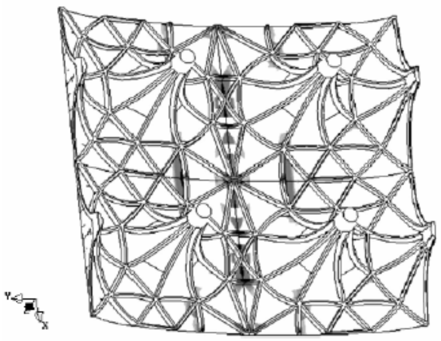

(e)

Figure 18. Structural analysis for Monastery of Jerónimos: (a) push-over analysis of compound; (b) model updating and dynamic time integration of church; (c) detailed analysis of nave; (d) results for the detailed analysis of the nave in terms of displacements and crack widths.

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